BROOKHAVEN
NATIONAL LABORATORY

BNL-78083-2007-CP

Methods for Addressing the Problem of the Dependence of the Time of Flight on Transverse Amplitude in Linear Non-Scaling FFAGs

J. Scott Berg

To Appear in the Proceedings of the 2006 FFAG Workshop, Osaka, Japan, 6–10 November 2006.

January 2007

Physics Department/Bldg. 901A

Brookhaven National Laboratory
P.O. Box 5000
Upton, NY 11973-5000
www.bnl.gov

DISCLAIMER

Printed on recycled paper

Methods for Addressing the Problem of the Dependence of the Time of Flight on Transverse Amplitude in Linear Non-Scaling FFAGs

J. Scott Berg

Brookhaven National Laboratory; Building 901A; P. O. Box 5000, Upton, NY 11973-5000

Abstract. Because the time of flight in a linear non-scaling FFAG depends on the transverse amplitude, motion in the longitudinal plane will be different for different transverse particle amplitudes. This effect, if not considered, will lead the failure of a substantial portion of the beam to be accelerated. I will first briefly review this effect. Then I will outline some techniques for addressing the problems created by the effect. In particular, I will discuss partially correcting the chromaticity and increasing the energy gain per cell. I will discuss potential problems with another technique, namely the introduction of higher harmonic cavities.

Keywords: Fixed Field Alternating Gradient Accelerator; Simulation; Tracking
PACS: 29.27.Bd

DEPENDENCE OF THE TIME OF FLIGHT ON TRANSVERSE AMPLITUDE

In most any machine, the time of flight will depend on the transverse amplitude. This arises because particles with large transverse amplitudes traverse a longer path than small amplitude particles do. The dependence of the time of flight on the transverse action is, to linear order, related to the chromaticity [1]. For acceleration in an FFAG where the energy gain does not depend on the time of flight (a sufficiently accurate assumption for estimating the effect), the time of flight difference between a particle with transverse normalized action $\vec{J}_n$ (in units of eV s) and a particle at zero transverse amplitude is, to lowest order in $\vec{J}_n$

$$-\frac{2\pi\Delta\vec{\nu}\cdot\vec{J}_n}{\Delta E}, \tag{1}$$

where $\Delta\vec{\nu}$ is the difference in tune per cell from the beginning to the end of acceleration, and ΔE is the energy gain per cell [1].

Equation 1 shows that there are three ways to reduce the effect of the dependence of the time of flight on transverse amplitude. The first is to reduce the transverse beam emittance. Accomplishing this can be expensive, and in any case will not be discussed further here, since modifying the accelerating FFAG design cannot reduce the emittance. The second method is to reduce the tune difference from the minimum to maximum energy. This is a form of chromaticity correction, and can be achieved by adding sextupole components to the magnets. The third method is to increase the energy gain per cell, which will require the addition of expensive RF cavities to the ring.

REDUCING TUNE VARIATION WITH ENERGY

There are two methods that one can use to reduce the tune variation with energy. The simplest is to reduce the average tune. This, unfortunately, results in a larger time of flight variation with energy [2], requiring more voltage in the ring to achieve the required longitudinal phase space acceptance [3]. It will also increase the magnet aperture, further increasing machine costs.

Instead, one can add sextupole components to the magnets of a linear non-scaling FFAG lattice to reduce the tune variation with energy. In a normal accelerator lattice with a small energy acceptance, one would choose the sextupole strengths to reduce the chromaticity at the reference energy. Because of the large range of energies in an FFAG, however, one must take a more global approach. Since the difference in tune from beginning to end is what is relevant, that is what I will attempt to modify. In the process, I will attempt to keep the tune profile similar to what it was without the sextupoles.

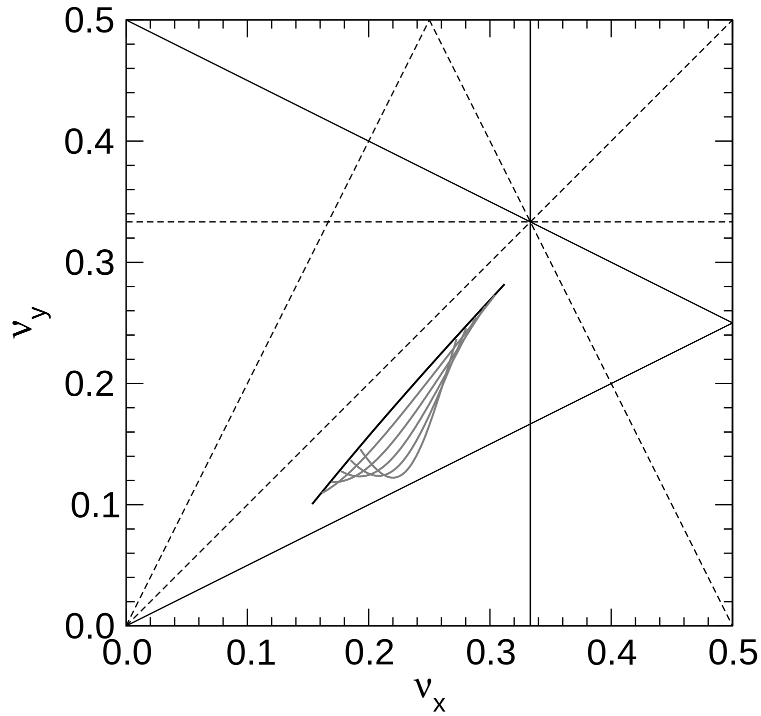

FIGURE 1. Tune variation with energy for varying amounts of sextupole. The nearly straight black line near the center of the diagram is for a lattice where the magnets have no sextupole components. The grey lines show the tune variation for lattice with sextupole components added to the magnets so as to reduce the tune variation from beginning to end by 10, 20, 30, 40, and 50%. The thinner lines that completely cross the diagram are resonance lines up to third order; the solid lines are directly driven by the sextupole components of the magnets.

I begin by choosing a lattice without sextupoles. Once the sextupoles are added, they will tend to drive third order resonances [4]. Thus, it is important to keep the tune away from the third order resonance lines. I will thus choose the tune so that the low energy tune is equidistant in the tune plane from the $3\nu_x = 1$ and $\nu_x - \nu_y = 0$, and the high energy tune is the same distance from the $\nu_x - 2\nu_y = 0$ line, as shown in Fig. 1.

In reducing the tune range of the lattice, I will keep the low and high energy tunes on a straight line between the low and high energy tunes of the lattice that doesn't have sextupoles. I will move both the high energy tune the low energy tune the same distance along that line. Thus,

$$\vec{\nu}_{\mathrm{lo}}(x) = (1-x/2)\vec{\nu}_{\mathrm{lo}}(0) + (x/2)\vec{\nu}_{\mathrm{hi}}(0) \qquad \vec{\nu}_{\mathrm{hi}}(x) = (x/2)\vec{\nu}_{\mathrm{lo}}(0) + (1-x/2)\vec{\nu}_{\mathrm{hi}}(0), \tag{2}$$

where x is the fraction by which $\Delta\vec{\nu}$ is reduced, $\vec{\nu}_{\mathrm{lo}}(0)$ is the low energy tune without sextupole, and $\vec{\nu}_{\mathrm{hi}}(0)$ is the high energy tune without sextupole. Figure 1 shows the resulting tune variation with energy. This requires not only adding a sextupole component to the magnets, but changing the dipole and quadrupole components as well. At some energies, the chromaticity is locally higher when the sextupole is added than when it is not present. At other energies it is lower. It is only $\Delta\vec{\nu}$ that is being reduced. One could probably add higher order multipole modes to the magnets to achieve a more local chromaticity reduction. However, this would directly drive higher order nonlinear resonances, meaning that one would have to avoid even more resonances than just the third order ones.

Linear non-scaling FFAGs are able to tolerate a large range of tunes because

- The lattice is made exclusively from simple, identical cells, so that in an error-free lattice, one need only be concerned with resonances of a single cell.
- The magnets are highly linear, meaning that any nonlinear resonances are only weakly driven.
- Any remaining resonances, from errors or residual nonlinearities, are accelerated through quickly.

To address the first point, when a sextupole component is added to the magnets, it is added in the same way in every cell. However, one cannot avoid departing from the second principle in adding a sextupole component: the magnets

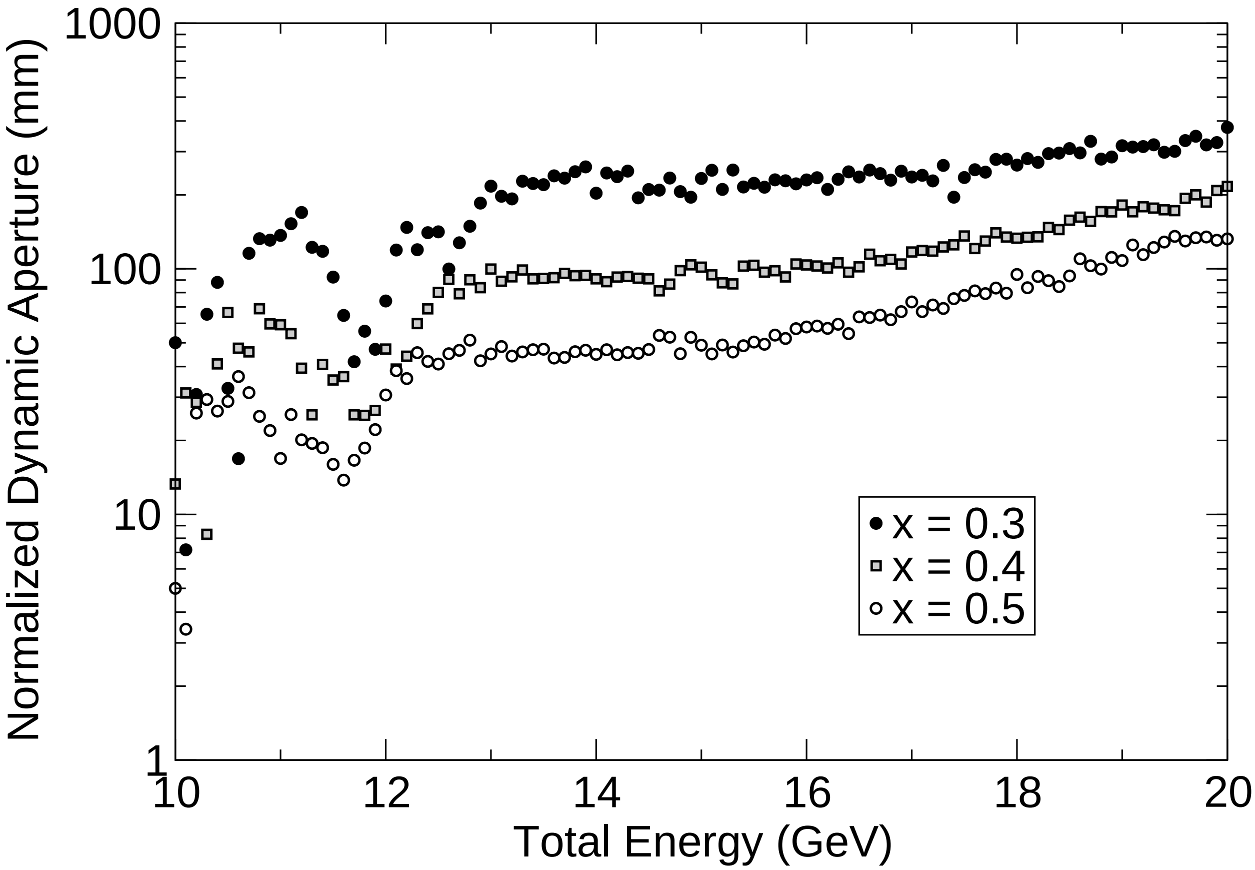

FIGURE 2. Dynamic aperture as a function of energy and x, the fractional reduction in the tune range for some of the lattices shown in Fig. 1. Results are similar to those obtained by Machida [5].

inevitably become nonlinear. This has the effect of reducing the dynamic aperture. One can hope that a modest amount of chromaticity correction, however, will still leave a sufficient dynamic aperture.

Figure 2 shows the dynamic aperture as a function of energy and the fractional reduction of $\Delta\vec{\nu}$ for some of the lattices shown in Fig. 1. The dynamic aperture was computed by beginning with a uniform ellipsoidal distribution of 100 particles in phase with the sum of action values less than a a given value, finding the smallest sum of actions of the particles that are lost (defined to be particles which get such large transverse momenta that the equations of motion become singular) within 200 cells, and iterating this procedure with that sum of action values until none of the 100 particles are lost.

For the $x = 0.5$ case, the values for 100 cells are nearly identical to the 200-cell values for energies above about 12.5 GeV. But below that energy, some energies have dynamic apertures as much as a factor of 2 lower in the 200-cell case compared to the 100-cell case (see Fig. 3). This is likely due to particles being lost near low-order resonances, such as the $4\nu_x = 1$ resonance. Similar results hold for other values of x.

Judging only from the higher energy values for the dynamic aperture, one might expect to be able to have very high values for x and still have acceptable dynamic aperture. One could even consider reducing the tune so as to keep it within the range required to have that large dynamic aperture, but this would come at a cost of increased magnet apertures and time of flight ranges [2]. Since these are 200-cell dynamic apertures, and acceleration is extremely rapid (accelerating from 10 to 20 GeV in 1000 to 1500 cells), one might be able to accelerate through some of the low-dynamic aperture energy ranges which are narrow without a significant impact on the beam.

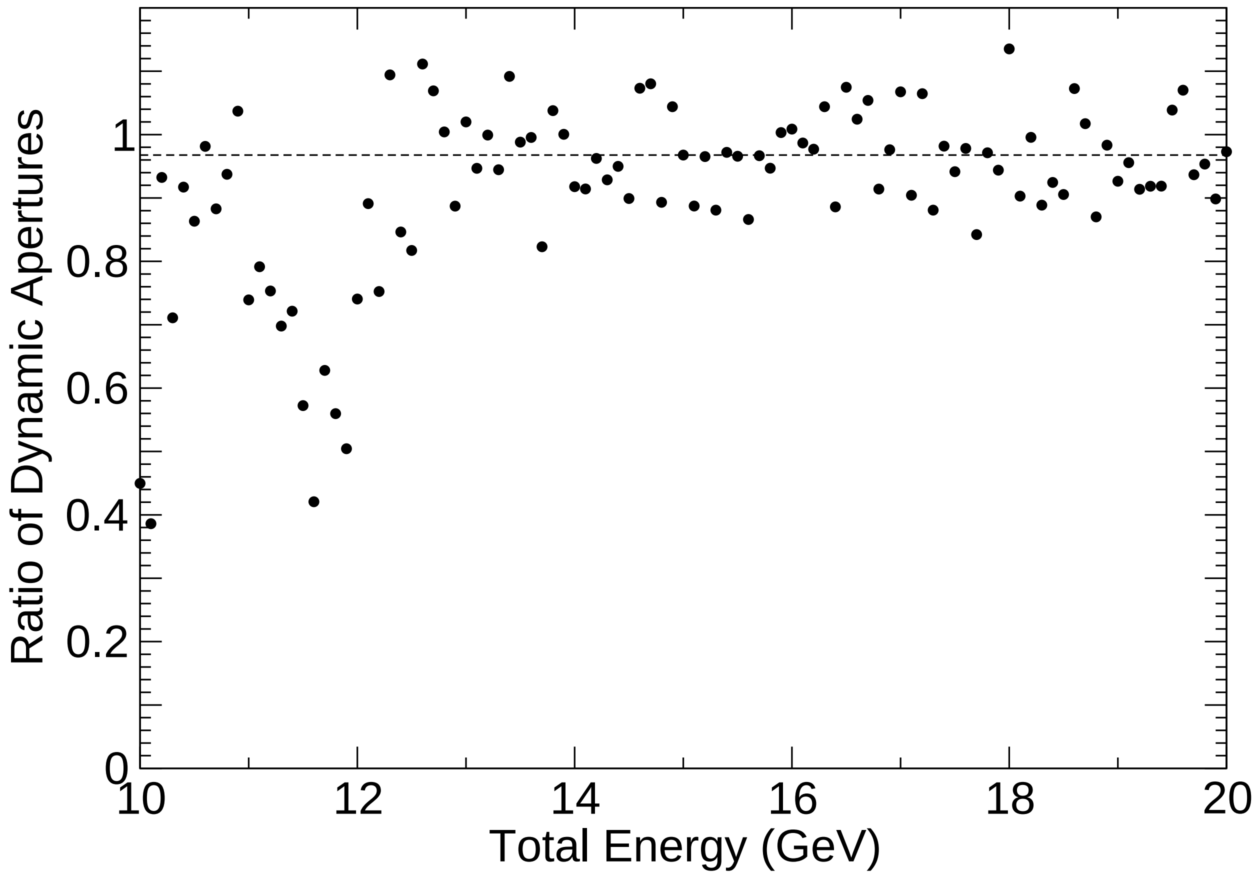

FIGURE 3. Ratio of the 200-cell dynamic aperture to the 100-cell dynamic aperture for the $x = 0.5$ case from Fig. 2. The dashed line is the average of the points with total energies greater than or equal to 12.5 GeV.

INCREASING THE ENERGY GAIN PER CELL

From Eq. 1, one can also reduce the time of flight variation with transverse amplitude by increasing the energy gain per cell.

Previous lattice designs [6] had left many cells without RF cavities. This made sense from a cost point of view, since for designs with a reasonable amount of decay, a longer ring actually is less expensive than a shorter one, since it can have a smaller magnet aperture and lower pole tip fields, and these cost reductions overcome the cost increase due to a larger number of magnets. Thus, one method of increasing the energy gain per cell is to fill every possible cell with an RF cavity. Similarly, instead of using one single-cell cavity in every cell, one can use a two-cell cavity in every cell.

Table 1 shows the results of constructing cost optimized lattices when filling every possible cell with one or two cavity cells. Filling every possible cell with a single RF cell increases the energy gain per cell by 31% and 49% for the 6.25–12.5 GeV and 12.5–25 GeV machines respectively, while only increasing their costs by 4% and 2%, respectively. By putting two cavity cells in each possible lattice cell, one can increase the energy gain per cell by almost a factor of 3, but with a increase in cost of 32% and 31%, respectively. Drift lengths must be adjusted to accommodate both cavity cells. In contrast to earlier results [6], FODO cells become modestly more cost-effective than doublet cells when two cavities per cell are used.

For the lower energy range, the number of turns that one can achieve in the FFAG when two cavities per cell are used is relatively small, and is becoming comparable to the number of linac passes one would make in a recirculating linear accelerator (RLA). This indicates that if one needs to have two cavities per cell to address the dependence of the time of flight on transverse amplitude, FFAGs at lower energies may not be more cost-effective than an RLA; one must carefully examine the cost estimates for RLAs and FFAGs to evaluate this.

TABLE 1. Basic lattice parameters and performance for several lattice configurations. Lattices are optimized for costs using techniques similar to [6]. For the "method," "Empty" indicates that the lattice is cost-optimized allowing an arbitrary number of cells without RF cavities. The other methods have exactly 6 cells without RF cavities. "1/Cell" indicates that there is one cavity cell per lattice cell, "2/Cell" indicates that there are two cavity cells per lattice cell. Cost is in relative units, and is based in the cost model in [7]. Costs include a reduction in decays from the "Empty" case [6]. ΔE is the energy gain per lattice cell.

Energy (GeV)	6.25–12.5				12.5–25			
Type	Doublet	Doublet	Doublet	FODO	Doublet	Doublet	Doublet	FODO
Method	Empty	1/Cell	2/Cell	2/Cell	Empty	1/Cell	2/Cell	2/Cell
Lattice Cells	69	61	50	51	93	78	63	64
Cavity Cells	48	55	88	90	58	72	114	116
Turns	10.8	9.3	5.8	5.7	18.2	14.6	9.2	9.0
Cost	80.7	82.3	116.8	115.5	95.0	98.7	140.2	135.8
ΔE (MV)	8.7	11.5	22.4	22.4	7.9	11.7	23.0	23.1

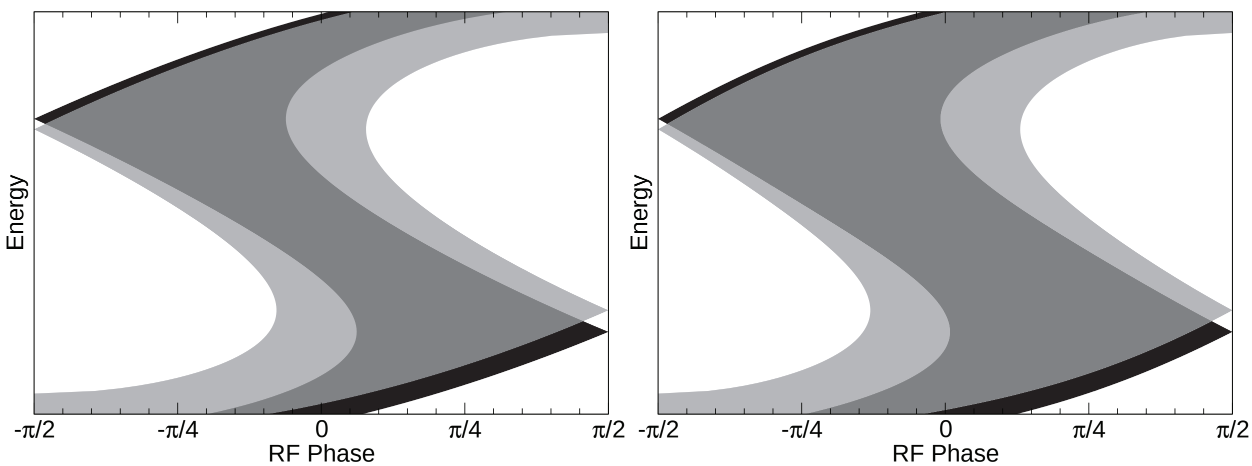

FIGURE 4. Longitudinal phase space for low transverse amplitude (light grey) and high transverse amplitude (black), showing region where they overlap (dark grey). Fundamental RF only is shown to the left, fundamental RF plus third harmonic RF, with the same maximum accelerating voltage as for the single harmonic, is shown to the right.

HIGHER HARMONIC RF CAVITIES

One can add higher harmonic cavities to an FFAG so as to reduce the dependence of energy gain on the time of flight. While this doesn't attack the problem directly, it attempts to reduce the negative impact of that phenomenon. It will increase the region of phase space that can be successfully accelerated for all transverse amplitudes (see Fig. 4), reduce the energy spread of the beam after acceleration, and reduce the distortion of the beam ellipse. The resulting beam, however, will still have a larger apparent longitudinal emittance, since even if the energy gain were independent of the time of flight, the time of flight spread, when integrated over the full transverse distribution, will be increased.

Higher harmonic RF cavities can, however, cause some problems. Since they have less stored energy than the main accelerating cavities, their voltage will drop significantly more quickly than that of the main RF cavities. This can be a significant problem in an FFAG with a large number of turns: the desired average RF wave form will not be maintained during the acceleration cycle. One could possibly attempt to keep the average wave form correct, but this would require even more higher harmonic voltage relative to the main RF voltage than one would require were the beam loading unimportant.

Furthermore, the space required for the higher harmonic cavities and the fact that their voltage opposes that of the main cavities means that the average energy gain per lattice cell will be reduced. As described in the previous section, this will make the dependence of the time of flight on transverse amplitude worse.

One must therefore carefully evaluate whether higher harmonic cavities will be beneficial for mitigating the effects of the dependence of the time of flight of transverse amplitude, taking into account the above considerations.

OTHER TECHNIQUES

There are other techniques which may reduce the effect of the dependence of the time of flight on transverse amplitude in non-scaling FFAGs. One of the most important is the choice of initial conditions. One should choose the phase space orientation of the beam, its initial phase, and the precise RF frequency (or equivalently the cell length) so as to optimize the phase space transmission over the entire range of transverse amplitudes we desire to accelerate. Work is progressing on how to determine these optimal conditions.

One should also attempt to place reasonable requirements on the systems upstream of these FFAGs. Lowering the incoming emittance is certainly the best way to address the dependence of time of flight on transverse amplitude. Furthermore, one should ideally specify that the FFAGs will transmit a distribution that is ellipsoidal in both longitudinal and transverse phase spaces: in other words, if one has an acceptance $A_\perp$ transversely and an acceptance $A_\parallel$ longitudinally, then

$$\frac{2J_\perp}{A_\perp} + \frac{2J_\parallel}{A_\parallel} < 1. \tag{3}$$

In other words, if the transverse amplitude ($2J_\perp$) is zero, one should get the full longitudinal acceptance; if, however, the transverse amplitude is $A_\perp$, then one should not expect any longitudinal transmission. Other systems in the machine which restrict acceptance are likely to create similar limitations, so this is not expected to give a significant reduction in performance.

CONCLUSIONS

The impact of the dependence of the time of flight on transverse amplitude in linear non-scaling FFAGs can be reduced by a number of methods. Reducing the tune variation with energy by adding nonlinearities to the magnets is effective, but reduces the dynamic aperture. A modest amount of nonlinearity can provide some improvement in the time of flight problem without reducing the dynamic aperture to unacceptable levels. Increasing the average accelerating voltage per cell can achieve modest improvements with little cost, and larger improvements at a significant cost. Higher harmonic cavities may also reduce the effects of the dependence of the time of flight on transverse amplitude, but may be problematic due to their lower stored energy and their reduction of the average accelerating gradient. Finally, one should look outside the FFAG itself, and consider reducing the incoming transverse emittance and choosing the initial distribution optimally.

ACKNOWLEDGMENTS

This work has been supported by the United States Department of Energy, Contract No. DE-AC02-98CH10886.

REFERENCES

1. J. S. Berg, *Nucl. Instrum. Methods A* **570**, 15–21 (2007).
2. J. S. Berg, and C. Johnstone, "Design of FFAGs Based on a FODO Lattice," in *Proceedings of the 2003 Particle Accelerator Conference*, edited by J. Chew, P. Lucas, and S. Webber, IEEE, Piscataway, NJ, 2003, pp. 2216–2218.
3. J. S. Berg, *Phys. Rev. ST Accel. Beams* **9**, 034001 (2006).
4. J. S. Berg, S. A. Bogacz, S. Caspi, J. Cobb, R. C. Fernow, J. C. Gallardo, S. Kahn, H. Kirk, D. Neuffer, R. Palmer, K. Paul, H. Witte, and M. Zisman, *Phys. Rev. ST Accel. Beams* **9**, 011001 (2006).
5. S. Machida, Dynamic aperture of non-scaling FFAG with sextupole (2006), URL `http://www.fnal.gov/projects/muon_collider/FridayMeetings/`, presentation at the NFMCC Friday Meeting, 21 July 2006.
6. J. S. Berg, "Recent Results from Optimization Studies of Linear Non-Scaling FFAGs for Muon Acceleration," in *The International Workshop on FFAG Accelerators, October 13–16, 2004, KEK, Tsukuba, Japan*, edited by S. Machida, Y. Mori, and T. Yokoi, 2005, pp. 1–8, URL `http://hadron.kek.jp/FFAG/FFAG04_HP/`, preprint NFMCC-doc-309-v1, available from `http://nfmcc-docdb.fnal.gov/`.
7. R. B. Palmer, and J. S. Berg, "A Model for Determining Dipole, Quadrupole, and Combined Function Magnet Costs," in *Proceedings of EPAC 2004, Lucerne, Switzerland*, EPAC, European Physical Society Accelerator Group, 2004, pp. 1807–1809.